THE IMPACT OF THEORETICAL UNCERTAINTIES IN THE HALO MASS FUNCTION AND HALO BIAS ON PRECISION COSMOLOGY

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ABSTRACT

We study the impact of theoretical uncertainty in the dark matter halo mass function and halo bias on dark energy constraints from imminent galaxy cluster surveys. We find that for an optical cluster survey like the Dark Energy Survey, the accuracy required on the predicted halo mass function to make it an insignificant source of error on dark energy parameters is $\approx 1\%$. The analogous requirement on the predicted halo bias is less stringent ($\approx 5\%$), particularly if the observable–mass distribution can be well constrained by other means. These requirements depend upon survey area but are relatively insensitive to survey depth. The most stringent requirements are likely to come from a survey over a significant fraction of the sky that aims to observe clusters down to relatively low mass, $M_{\rm th} \approx 10^{13.7} \, h^{-1} M_{\odot}$; for such a survey, the mass function and halo bias must be predicted to accuracies of $\approx 0.5\%$ and $\approx 1\%$, respectively. These accuracies represent a limit on the practical need to calibrate ever more accurate halo mass and bias functions. We find that improving predictions for the mass function in the low-redshift and low-mass regimes is the most effective way to improve dark energy constraints.

Subject headings: cosmological parameters — cosmology: theory — galaxies: clusters — galaxies: halos

1. INTRODUCTION

Observations of the number density of galaxy clusters as a function of cluster mass and redshift are powerful probes of cosmology, especially the accelerated cosmological expansion caused by the cryptically dubbed *dark energy*. Cosmological parameters have been recently inferred from a number of observations employing different techniques for cluster identification (e.g., Gladders et al. 2007; Mantz et al. 2008; Henry et al. 2009; Rozo et al. 2010; Vikhlinin et al. 2009; Mantz et al. 2009b). The effort to derive precise and unbiased constraints on cosmological parameters, particularly those describing dark energy, requires control of various systematic error sources. In this paper, we estimate the precision with which the abundance of dark matter halos as a function of mass (the halo *mass function*) and the clustering of halos as a function of mass (the halo *bias*) must be predicted in order to ensure that errors in the predictions for these quantities will be a small fraction of the error budget in forthcoming cluster count cosmology efforts.

For the current generation of cluster count surveys, such as the Sloan Digital Sky Survey (SDSS), the Red-Sequence Cluster Survey (RCS), and the Massive Cluster Survey (MACS), uncertainty in the mass function is unlikely a major concern. The dominant errors in contemporary surveys are thought to be either limited statistics (e.g., Mantz et al. 2008, for MACS) or the uncertain relation between observable quantities and cluster mass (e.g., Gladders et al. 2007; Rozo et al. 2010, for RCS and SDSS respectively). Uncertainty in the relation between cluster observables and masses will be an important limitation (e.g., Majumdar & Mohr 2003, 2004; Lima & Hu 2004, 2005; Stanek et al. 2006; Lima & Hu 2007; Cunha 2009), and it is thought that some combination of empirical and theoretical insight into the observable–mass

relation will continue to prove effective in the interpretation of forthcoming data (e.g., Kravtsov et al. 2006; Rozo et al. 2009). Understanding the relation between observables and mass is a rapidly evolving subject, so it is difficult to anticipate the level at which this will be controlled in the analyses of forthcoming data. Although we do not directly deal with observable–mass relations, we explore the dependence of our results on assumptions about the observable–mass relation.

The motivations to study the uncertainty in predicting halo abundances and clustering are twofold. First, forthcoming optical surveys, such as the Dark Energy Survey (DES)[3], the Panoramic Survey Telescope & Rapid Response System (PanSTARRS)[4], and the Large Synoptic Survey Telescope (LSST)[5]; X-ray surveys, such as the extended ROentgen Survey with an Imaging Telescope Array (eRosita)[6]; and Sunyaev–Zeldovich (SZ) surveys, such as the Atacama Cosmology Telescope (ACT)[7] and the South Pole Telescope (SPT)[8], all promise to expand greatly upon contemporary observations of clusters. These surveys will enable significant reductions in statistical errors and provide better constraints on the cluster observable–mass distribution. With ever-improving theoretical understanding of this distribution, systematic errors that are currently unimportant may be damaging to future surveys and need to be controlled. For example, Crocce et al. (2010) demonstrated that current errors in predicted mass functions may lead to statistically significant systematic errors in the inferred dark energy equation of state from SZ surveys.

Second, there is a significant, ongoing effort to develop ever more accurate predictions for the halo mass function (e.g., Sheth & Tormen 1999; Sheth et al. 2001; Jenkins et al. 2001; Evrard et al. 2002; Reed et al. 2003; Warren et al.

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[3] http://www.darkenergysurvey.org
[4] http://pan-starrs.ifa.hawaii.edu
[5] http://www.lsst.org
[6] http://www.mpe.mpg.de/projects.html#erosita
[7] http://www.physics.princeton.edu/act/
[8] http://pole.uchicago.edu/

2006; Lukić et al. 2007; Cohn & White 2008; Tinker et al. 2008; Robertson et al. 2009; Lukić et al. 2009; Crocce et al. 2010) and halo bias (e.g., Sheth & Tormen 1999; Sheth et al. 2001; Seljak & Warren 2004; Tinker et al. 2008, 2010). Mass functions and halo biases have been derived on theoretical grounds (Press & Schechter 1974; Bond et al. 1991; Mo & White 1996; Sheth & Tormen 1999; Sheth et al. 2001; Maggiore & Riotto 2010, 2009a,b, see Zentner 2007 for a recent review). Fits to numerical simulations with theoretically motivated functional forms have been provided by several groups, and the current state-of-the-art includes the recent papers by Lukić et al. (2007), Tinker et al. (2008), and Crocce et al. (2010), as well as the forthcoming work of the Los Alamos National Laboratory group (S. Bhattacharya et al. 2010, in preparation). Current theoretical predictions of the mass function are accurate at the $\sim 10\% - 30\%$ level (e.g., Tinker et al. 2008; Robertson et al. 2009; Rudd et al. 2008; Stanek et al. 2009), and future efforts will continue to improve this accuracy. These efforts require numerous cosmological numerical simulations along with requisite verification, validation, and considerable analysis, involving large commitments of both computational and human resources. Therefore, establishing the accuracy in the mass function and halo bias that are required by surveys is a particularly timely issue.

In this work, we quantify how accurately halo mass functions and biases need to be predicted in order to have a negligible contribution to the error budgets of forthcoming cluster surveys. We begin in Section 2 with a simple demonstration of the potential errors that may be induced by the inaccuracy in the mass function. We present our mass function parameterizations and cosmological parameter constraint forecasts in Section 3.1 and Section 3.2, respectively. In all cases, we take the Tinker et al. (2008) mass function and Tinker et al. (2010) halo bias as a fiducial model, about which we perturb to estimate errors relevant to forthcoming optical and SZ surveys.

We detail our results in Section 4. In summary, we find that for relatively near-term optical surveys like DES, or SZ surveys like SPT, the mass function must be calibrated to the $\approx 1\%$ level while the halo bias must be calibrated to the $\approx 5\%$ level. The requirement on the mass function is relatively insensitive to different assumptions about the observable–mass distributions within the range considered in contemporary work, whereas the requirement on the halo bias is more sensitive. For longer-term experiments, such as a nearly half-sky optical survey planned for LSST, the calibration must be improved to the $\approx 0.5\%$ level in the mass function and the $\approx 1\%$ level in the halo bias. This represents the most stringent requirement for the surveys being planned over the next decade and may serve as a practical endgame in the need to refine theoretical predictions for halo abundances and clustering.

We also explore the halo masses and redshifts at which it is most important to make accurate predictions for the halo mass function. We find that the most effective strategy to improve dark energy constraints is to improve predictions at the low masses and low redshifts involved in the survey. Other details of the dependence of calibration requirements on mass and redshift depend upon the observable–mass distribution. We summarize our results and draw our conclusions in Section 5.

As we were completing our study, we learned of the related work of Cunha & Evrard (2009). Although our results generally agree with the results in this study, our study has a number of distinct and complementary aspects. These authors concentrated on the specific parameters in the Tinker et al. (2008)
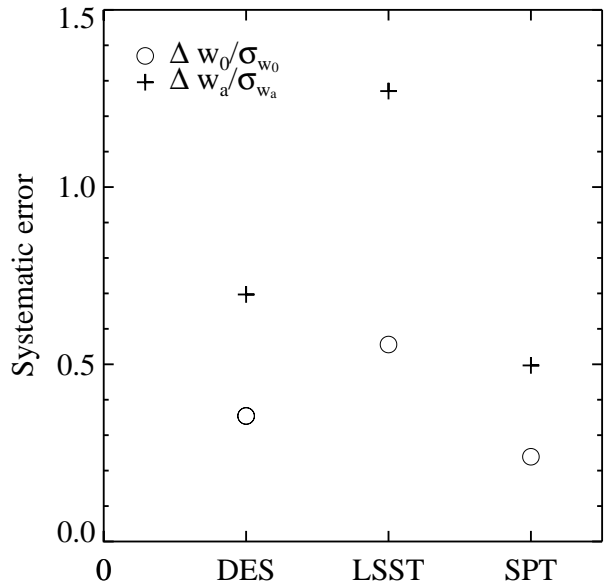

FIG. 1.— Potential systematic errors in dark energy parameter estimates caused by inaccurate modeling of the mass function. The systematic errors are shown in units of the statistical uncertainties expected from each survey. We assume that the fitting formula in Tinker et al. (2008) describes the data, while that in Sheth & Tormen (1999) is used in the likelihood function. The effect on imminent cluster surveys is not negligible, especially for a wide-area survey like LSST. Moreover, this systematic error is representative of current limitations of N-body methods, while baryonic processes are known to cause shifts in halo mass functions of equal or greater magnitude (Rudd et al. 2008; Stanek et al. 2009).

mass function and the Sheth & Tormen (1999) halo bias, focusing considerable study on the degeneracies between nuisance parameters and cosmological parameters. In contrast, we aim to describe the uncertainty in mass function and halo bias in a manner that is independent of the form of any specific fitting function. We discretize the mass function and halo bias, and assign to each mass and redshift bin a distinct nuisance parameter specifying the fractional deviation of the mass function or halo bias from the fiducial values. This parameterization also allows us to identify the masses and redshifts at which it is most fruitful to refine predictions in order to limit systematic errors on dark energy parameters.

2. THE EFFECT OF INACCURACY IN THE HALO MASS FUNCTION

We begin with an example showing the influence of inaccurate modeling of the halo mass function on the cosmological parameters derived from galaxy cluster surveys. We compare two commonly used fitting functions derived from N-body simulations, the Sheth & Tormen (1999) and Tinker et al. (2008) mass functions. To construct a concrete example, we assume that the Tinker et al. (2008) function describes the true halo mass function, while the Sheth & Tormen (1999) function serves as an example of an imperfect model in our likelihood analysis. The differences between these functions are $\sim 20\%$ over the most relevant masses (see Tinker et al. 2008). This represents a reasonable assessment of our current ability to predict the mass function. For example, Tinker et al. (2008) and Robertson et al. (2009) discussed fundamental limitations in the ability of contemporary methods to construct analytical mass function fits at the $\sim 10\%$ level, and baryonic processes can give rise to additional deviations in predicted mass func-

tions as large as $\sim 30\%$ (Rudd et al. 2008; Stanek et al. 2009).

We use the Fisher matrix formalism for parameter constraint forecasting to compute the systematic error in dark energy parameters due to a mass function that is imperfectly calibrated. We use the term "systematic error" to describe the offset between inferred values of cosmological parameters and the true values of these parameters. This systematic error is often referred to as a "bias" in the literature, but we do not use this term in order to avoid confusion with the halo bias. We refer the reader to the appendix of Wu et al. (2008) for the derivation of the systematic errors relevant for cluster surveys and defer further discussion of our methods to the following sections. We describe dark energy by a two-parameter equation of state $w(a) = w_0 + (1-a)w_a$ and take w_0 and w_a as free parameters.

Figure 1 shows the systematic errors in dark energy parameters in terms of the statistical uncertainties. We present the absolute value of the error, and zero indicates that there is no systematic error. Three survey assumptions are shown: a DES-like optical survey ($5000\,\mathrm{deg}^2$ to a maximum redshift $z_{\mathrm{max}} = 1$), an LSST-like optical survey ($20,000\,\mathrm{deg}^2$ to $z_{\mathrm{max}} = 1$), and an SPT-like SZ survey ($2000\,\mathrm{deg}^2$ to $z_{\mathrm{max}} = 1.5$). As can be seen, the dark energy equation of state parameters may have systematic errors at levels of approximately 20%–130% of their statistical uncertainties. These errors indicate that current inaccuracy in the halo mass function would not be a negligible contribution to the errors in dark energy parameters inferred from the next generation of surveys. In fact, this is likely a conservative estimate of the systematic error because it does not include additional uncertainties induced by baryonic physics.

3. METHODS

3.1. Parameterizing Uncertainty in the Halo Mass Function and Halo Bias

In this subsection, we describe our parameterization of an uncertain halo mass function $dn/d\ln M$ and an uncertain halo bias $b(M)$, aiming to decouple our results from the particulars of published fits to halo mass functions and biases. For the mass function, we define a set of nuisance parameters f_i as follows. We take the ratio of the actual mass function to a fiducial mass function at mass M and redshift z to be

$$\frac{dn/d\ln M}{(dn/d\ln M)_{\mathrm{Fid}}} = f_i \psi_i(M,z) \; , \tag{1}$$

where the $\psi_i(M,z)$ describes the binning in mass and redshift and equals unity when (M,z) is within the range specified for bin i. The index i runs over all bins in both the mass and redshift dimensions.

As we will discuss in more detail in Section 3.2, galaxy cluster surveys observe halo mass proxies above some observable threshold. In what follows, we model the theoretical mass function down to a minimum mass that is lower than the observable threshold by an amount comparable to the scatter in the observable–mass distribution ($\sigma_{\ln M}$, which will be defined later). This low minimum mass is necessary to account for the scatter, which may lead relatively low-mass clusters with higher-than-average mass proxies to be included in an observed cluster sample.

Between the minimum mass and a maximum mass, we use several bins for the theoretical mass function. We choose the width of the bins to be roughly comparable to the scatter in

the relation between observable and mass. This scheme follows from two considerations. First, the binning in the theoretical mass function should be sufficiently fine so that constraints on dark energy are maximally degraded when there is no prior information on any of the f_i nuisance parameters. In this way correlated shifts in the mass function over a range of masses do not significantly aid in self-calibration. Second, very fine mass bins are unnecessary because the scatter in the observable–mass relation gives an effective resolution with which the mass function may be probed. Binning significantly more finely means that several theoretical mass bins contribute to a measurement at a particular observable mass proxy. The value of the mass function in each bin will then need to be predicted with less accuracy, but the combination that results in the measurement will need to be predicted with fixed accuracy.

Following from these considerations, it is clear that mass function accuracy requirements should depend upon bin size. We quote results that are realized when the mass function is binned at a resolution comparable to the scatter in the relation between observable and mass. This seems sensible in that it allows for maximal degradation of cosmological constraints when the mass function is uncertain, but does not sample the mass function significantly more finely than observations.

For our model optical surveys, this choice amounts to five bins spaced evenly in $\ln M$ between $M_{\mathrm{min}} = 10^{13.3}\ h^{-1} M_\odot$ and $M_{\mathrm{max}} = 10^{15.3}\ h^{-1} M_\odot$. For our model SZ surveys, we use seven mass bins above $M_{\mathrm{min}} = 10^{13.9}\ h^{-1} M_\odot$. Note that these binning methods for $\ln M$ include one bin below the threshold of observable mass proxies discussed in the next section. We bin in redshift intervals of $\Delta z = 0.1$ from $z = 0$ to a maximum survey redshift, z_{max}. For our fiducial mass function, we use the mass function fit at a spherical overdensity of $\Delta = 200$ in Tinker et al. (2008).

We introduce an analogous set of parameters g_i to describe the uncertainty in halo bias around the fiducial value from Tinker et al. (2010),

$$\frac{b(M,z)}{b_{\mathrm{Fid}}(M,z)} = g_i \psi_i(M,z) \; , \tag{2}$$

according to a binning scheme identical to that of the mass function. The halo bias and mass function may well be linked based on physical considerations (Kaiser 1984; Mo & White 1996; Sheth & Tormen 2002; Zentner 2007, e.g., through the excursion set approach), but we treat them independently as a conservative way to account for the inadequacy of the assumptions of particular models.

In our statistical forecasts, we include the parameters f_i and g_i at each of the set mass and redshift bins. We then determine how well we must be able to set priors on the parameters f_i and g_i. We refer to the prior knowledge of the f_i and g_i as $\sigma_{\mathrm{f_i}}$ and $\sigma_{\mathrm{g_i}}$ respectively.

3.2. Implementation

We forecast constraints on cosmological parameters using the methods described in detail in Wu et al. (2008, 2009). In what follows, we give a brief description of the aspects of this calculation necessary to quantify the effect of uncertainty in the mass function and halo bias and refer the reader to the aforementioned papers for a more complete description.

We assume that a survey determines a particular mass proxy M_{obs} and that the survey clusters are binned in $\ln M_{\mathrm{obs}}$ according to a binning function $\phi(\ln M_{\mathrm{obs}})$. We assume that mass

proxies are related to mass through an observable–mass distribution $P(\ln M_{obs}|\ln M)$ which specifies the probability that a halo with natural logarithm of mass $\ln M$ gives a mass proxy $\ln M_{obs}$. As an example, consider a survey composed of a number of cells of volume V, each of which spans a narrow range in redshift. The mean number of clusters in each cell is

$$\bar{m} = V \int d\ln M \left(\frac{dn}{d\ln M}\right) \langle\phi|\ln M\rangle$$
$$= V \sum_k \int d\ln M f_k \psi_k(M, z) \left(\frac{dn}{d\ln M}\right)_{Fid} \langle\phi|\ln M\rangle \quad (3)$$

where

$$\langle\phi|\ln M\rangle \equiv \int d\ln M_{obs} P(\ln M_{obs}|\ln M)\phi(\ln M_{obs}). \quad (4)$$

and the index k runs over all mass bins in the theoretical mass function at the corresponding redshift.

We apply counts-in-cells and self-calibration as in Lima & Hu (2004, 2005), calculating both the expected counts in different bins, $\bar{\mathbf{m}}$, and the variance in counts, $\mathbf{S} = \langle(\mathbf{m} - \bar{\mathbf{m}})^T(\mathbf{m} - \bar{\mathbf{m}})\rangle$. The Fisher matrix for a cluster survey is given by

$$F_{\alpha\beta} = \bar{\mathbf{m}}_{,\alpha}^T \mathbf{C}^{-1} \bar{\mathbf{m}}_{,\beta} + \frac{1}{2}\text{Tr}[\mathbf{C}^{-1}\mathbf{S}_{,\alpha}\mathbf{C}^{-1}\mathbf{S}_{,\beta}], \quad (5)$$

where $\mathbf{C} = \text{diag}(\bar{\mathbf{m}}) + \mathbf{S}$ is the total covariance matrix. The comma followed by a Greek letter subscript refers to the derivatives with respect to model parameters, which include multiple f_i and g_i in this analysis. With this notation, the derivative of the mean cluster count in a particular cell with respect to f_i reads

$$\bar{m}_{,f_i} = \frac{\partial \bar{m}}{\partial f_i} = V \int d\ln M \psi_i(M) \left(\frac{dn}{d\ln M}\right)_{Fid} \langle\phi|\ln M\rangle. \quad (6)$$

Analogous expressions hold for the variance in counts.

We assume an observable–mass distribution $P(\ln M_{obs}|\ln M)$ that is a Gaussian with a mean $\langle\ln M_{obs}|\ln M\rangle = \ln M + \ln M_{bias}$ and a variance $\sigma_{\ln M}^2$. We parameterize the mean and variance of our observable–mass distribution as in Lima & Hu (2005), with an additional mass dependence:

$$\ln M_{bias} = \ln M_0 + \alpha_M \ln(M/M_{pivot}) + \alpha_z \ln(1 + z)$$
$$\sigma_{\ln M}^2 = \sigma_{Fid}^2 + \beta_M \ln(M/M_{pivot}) + B_0 + B_1 z + B_2 z^2 + B_3 z^3 \quad (7)$$

Here M_{pivot} is the pivot mass characterizing the mass dependence. We fix its value to the observable threshold of the survey and note that the uncertainty in M_{pivot} can be accounted for with B_0 and $\ln M_0$ so that this choice is innocuous. The parameter $\sigma_{\ln M}$ is referred to as the observable–mass scatter throughout this work. This model includes eight nuisance parameters in total: $(\ln M_0, \alpha_M, \alpha_z, \beta_M, B_0, B_1, B_2, B_3)$; their fiducial values are all assumed to be zero.

We specify different surveys by an observable threshold M_{th}, a characteristic choice of observable–mass distribution, a maximum survey redshift z_{max}, and a survey area A. For each survey assumption, we choose a bin size that is narrow enough to ensure no loss of information due to binning, and we test the convergence of parameter constraints with bin size. We consider two broad classes of survey that differ in the method by which clusters are identified. First we consider typical "optical cluster survey" parameters with a relatively low observable threshold and a relatively broad dispersion in the observable–mass distribution. Second, we consider typical "SZ cluster survey" parameters with a relatively high observable threshold but a comparably small observable–mass dispersion. The details of these classes are as follows.

1. Optically selected clusters: $\sigma_{Fid} = 0.4$, $M_{th} = 10^{13.7} h^{-1} M_\odot$, 8 bins in M_{obs} with width $\Delta \log_{10} M_{obs} = 0.2$, and $z_{max} = 1$.

2. SZ-selected clusters: $\sigma_{Fid} = 0.2$, $M_{th} = 10^{14.1} h^{-1} M_\odot$, 12 bins in M_{obs} with width $\Delta \log_{10} M_{obs} = 0.1$, and $z_{max} = 1.5$.

We specifically consider a "DES-like" survey of optically selected clusters over $5000 \deg^2$ of sky and an "SPT-like" survey of SZ-selected clusters over $2000 \deg^2$. We will discuss results relevant for X-ray surveys, but we do not present distinct calculations for the X-ray case. As we will describe in the next section, we find that X-ray surveys drive requirements similar to or less stringent than those of SZ surveys.

We characterize the statistical power of galaxy cluster surveys using the figure of merit (FoM) proposed in the Report of the Dark Energy Task Force (DETF; Albrecht et al. 2006):

$$\text{FoM} = 1/\sqrt{\det Cov(w_0, w_a)} = [\sigma(w_a)\sigma(w_p)]^{-1}, \quad (8)$$

where $w = w_0 + (1 - a)w_a$, and w_p is calculated at the pivot redshift at which w is best constrained. We assume a fiducial cosmology given by the WMAP5 best-fit (Komatsu et al. 2009) with parameters: $w_0 = -1$, $w_a = 0$, $\Omega_{DE} = 0.726$, $\Omega_k = 0$, $\Omega_m h^2 = 0.136$, $\Omega_b h^2 = 0.0227$, $n_s = 0.960$, $\Delta_\zeta = 4.54 \times 10^{-5}$ at $k = 0.05 \text{Mpc}^{-1}$. We use a Planck prior Fisher matrix provided by Z. Ma and W. Hu (2008, private communication). This cosmic microwave background (CMB) information is included in all of our calculations.

In what follows, we quote our results relative to the FoMs that may be attained in the limit of perfect predictions for the mass and bias functions of halos. For each survey, the FoM in this limit can span a range between a fixed observable–mass relation and an observable–mass relation in which all eight nuisance parameters must be self-calibrated. For a DES-like survey, these baseline figures of merit are FoM = 17.1 (self-calibrated observable–mass relation) and FoM = 139 (fixed observable–mass relation). In the case of an SPT-like survey, the baseline figures of merit range from FoM = 5.7 (self-calibrated observable–mass relation) to FoM = 42.4 (fixed observable–mass relation).

4. RESULTS

4.1. General Mass Function and Halo Bias Requirements

We begin by calculating the degradation in dark energy constraints due to uncertainty in the mass function and halo bias. Figure 2 shows the ratio of the degraded FoM with respect to the fiducial FoM with perfectly known mass function and halo bias. We assume that all of the f_i (see Equation (1)) have the same prior, σ_f, and all of the g_i (see Equation (2)) have the same prior, σ_g. The left panels correspond to a DES-like survey, and the right panels correspond to an SPT-like survey.

The top panels of Figure 2 assume no prior constraints on the observable–mass distribution. For a DES-like survey, less than 10% degradation in the FoM requires about percent-level precision on the mass function and halo bias. Note that the FoM is only fractionally degraded when the mass function
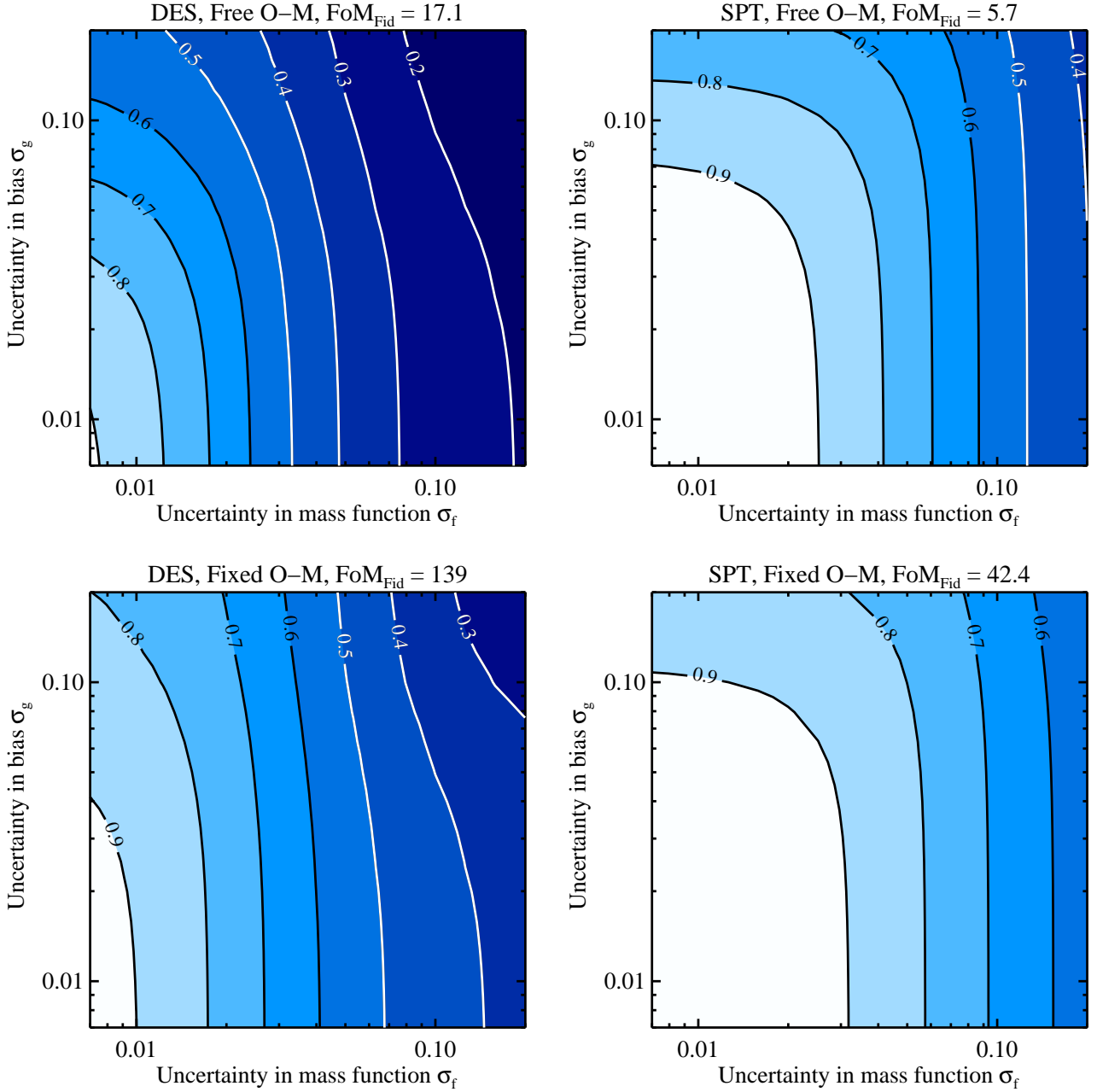
FIG. 2.— Degradation of the FoM due to uncertainties in mass function and halo bias. The contours and numbers correspond to the degraded FoM with respect to the FoM with perfectly-known mass function and halo bias. In order to quote a relatively simple result, we assume that the mass function parameters all have the same prior, σ_f, and that the halo bias parameters all have the same prior, σ_g. Given the statistical power of DES- and SPT-like surveys, the mass function needs to be predicted with a few percent precision to avoid 10% degradation in the FoM. For a perfectly known observable–mass distribution, the required precision of halo bias is less stringent because the information from sample variance becomes less important. Comparing a DES-like survey and an SPT-like survey, the latter has less stringent requirements because its smaller sky coverage and higher observable threshold result in fewer observed clusters.

uncertainties increase by an order of magnitude. For example, when the uncertainty is as high as 10%, the FoM degradation is approximately 70%. On the other hand, for an SPT-like survey, the FoM is less sensitive to mass function and halo bias uncertainties. This is because an SPT-like survey has a higher observable threshold and smaller sky coverage so that the expected cluster counts are lower and the *statistical* errors are larger.

The bottom panels of Figure 2 assume that the observable–mass distribution is perfectly constrained. Under this assumption, the requirement for mass function predictions is slightly less stringent than the case of a free observable–mass distribution shown in the top panels. However, the requirement for the halo bias predictions is significantly less strict. This behavior stems from including the information from sample variance for self-calibrating the observable–mass distribution. When
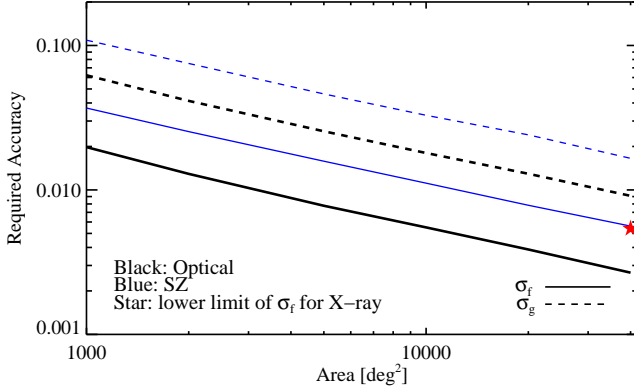
FIG. 3.— Effect of survey area on the required accuracy of the mass function and halo bias. We show the required precision in f_i and g_i that can avoid 10% degradation in the FoM. The required precision is inversely proportional to square root of the survey area. In the context of our models, optical surveys require higher accuracy than SZ surveys because they yield larger cluster counts and thus have smaller *statistical* errors. We find that this result is fairly insensitive to the maximum depth of the survey and the assumptions of the observable–mass distribution. We mark as a star the estimated lower limit of the required σ_f for a mass proxy with 5% scatter, which is relevant to X-ray surveys.

the observable–mass distribution is uncertain, both the mass function and halo bias need to be well known to avoid degrading the power of self-calibration. On the other hand, if the observable–mass distribution is well known, the requirement on the halo bias is markedly less stringent because the variance in counts is no longer needed to calibrate this relation. Nevertheless, the small statistical errors on dark energy in the case of a well-known observable–mass distribution mean that stringent mass function predictions are still necessary.

We note that although the top panels and the bottom panels show similar fractional degradations in the FoM, they have very different FoM values and thus very different absolute degradations. When the observable–mass distribution is well known, uncertainties in the mass function and halo bias will become the obstacle to achieve precision cosmology. On the other hand, when the observable–mass distribution is unknown, constraining the observable–mass distribution will be more effective to improve the FoM than constraining the mass function and halo bias. For detailed comparisons of uncertainties in the observable–mass distribution and those in the mass function and halo bias, we refer the reader to Figures 2 and 3 in Cunha & Evrard (2009).

It is worth reiterating that we have treated the mass function and halo bias as completely independent, despite physical considerations by which these quantities should be linked. For example, Manera et al. (2010) have shown that the peak-background split approach (Kaiser 1984; Mo & White 1996; Sheth & Tormen 2002) can be used to predict halo biases to $\sim 5\%$ given a halo mass function. Comparing this to the results of Figure 2, an SPT-like survey can tolerate a $\sim 5\%$ uncertainty in the halo bias when the mass function is known, for all practical purposes, perfectly. This indicates that it is not necessary to consider halo bias as an independent set of nuisance parameters in this case. Even the requirements of DES are not very far from this $\sim 5\%$ level of precision, suggesting that large degradations due to an uncertain halo bias when the mass function is well known are unlikely to happen.

In Figure 3, we show the dependence of calibration requirements on survey area. To be specific, we calculate the required σ_f and σ_g that correspond to a 10% degradation in the FoM at each value of survey area A. In cases where we compute the required σ_f, we assume perfect knowledge of the halo bias; we assume the converse when computing the required σ_g. For the results shown in Figure 3, we have assumed that there are no priors on the nuisance parameters of the observable–mass distribution.

The required σ_f depends upon the survey area A roughly as $\sigma_f \propto 1/\sqrt{A}$ for the following reason. In the Fisher matrix, the information from the data scales as A, and the information from priors scales as σ_f^{-2}; therefore, the scaling of σ_f is expected from requiring comparable information from both data and priors. The same reasoning applies for σ_g. When comparing optical and SZ surveys, we can see that SZ surveys require less accuracy in the mass function and halo bias at fixed sky coverage because of the higher observable thresholds.

We find this result to be relatively insensitive to the maximum survey redshift once $z_{max} \gtrsim 0.6$. Above this redshift, when we include higher-redshift bins, the increase of dark energy information is not as rapid as in the case of low redshift. In addition, when redshift increases, we introduce more nuisance parameters, and these parameters need no higher precision than those at low redshift. We also find that the results of Figure 3 depend only weakly on the scatter and the assumed priors for the observable–mass relation over a wide range of reasonable values.

We do not explicitly explore the requirement for an X-ray cluster survey because it can be estimated from our SZ results by considering the scatter and observable threshold. Since X-ray clusters tend to have smaller scatter (e.g. Mantz et al. 2009a) than SZ clusters, we calculate the case of a full-sky survey with a scatter 0.05 and a constant observable threshold $M_{th} = 10^{14.1}\ h^{-1}M_\odot$. The requirement on f_i is about 0.5%, which is very close to the SZ result. We mark this estimate as a star in Figure 3. On the other hand, X-ray clusters tend to have a higher observable threshold than SZ clusters at high redshift. As we have shown, higher observable threshold requires less stringent constraints. Therefore, we claim that the requirement for a full-sky X-ray cluster survey will be less stringent than 0.5%.

4.2. *Comparing Bins*

Next, we address the relative importance of the accuracy in the predictions for the halo mass function at different masses and redshifts. There are a number of methods that could be used to describe the relative importance of different halo masses and redshifts. We quantify the relative importance as follows. We begin with a very nonrestrictive prior on the mass function at all masses and redshifts. We then choose a single f_i and tighten the prior on this individual parameter. Subsequently, we compute the percentage improvement in the dark energy FoM with the more restrictive prior on the mass function in the single bin. We repeat this procedure for all mass and redshift bins and display the relative FoM improvements in Figure 4. In each panel, the horizontal axis represents redshift bins, and the vertical axis represents mass bins. For a DES-like optical survey (left panels), we start with a 10% prior on f_i and improve it to 1%; for an SPT-like SZ survey (right panels), we start with a 30% prior and improve to 3%.

The relative importance of different bins does depend upon the assumptions of the observable–mass distribution. We show the results for three different cases in Figure 4. In the top panels, we assume a fixed observable–mass relation. In the middle panels, we assume an observable–mass relation in
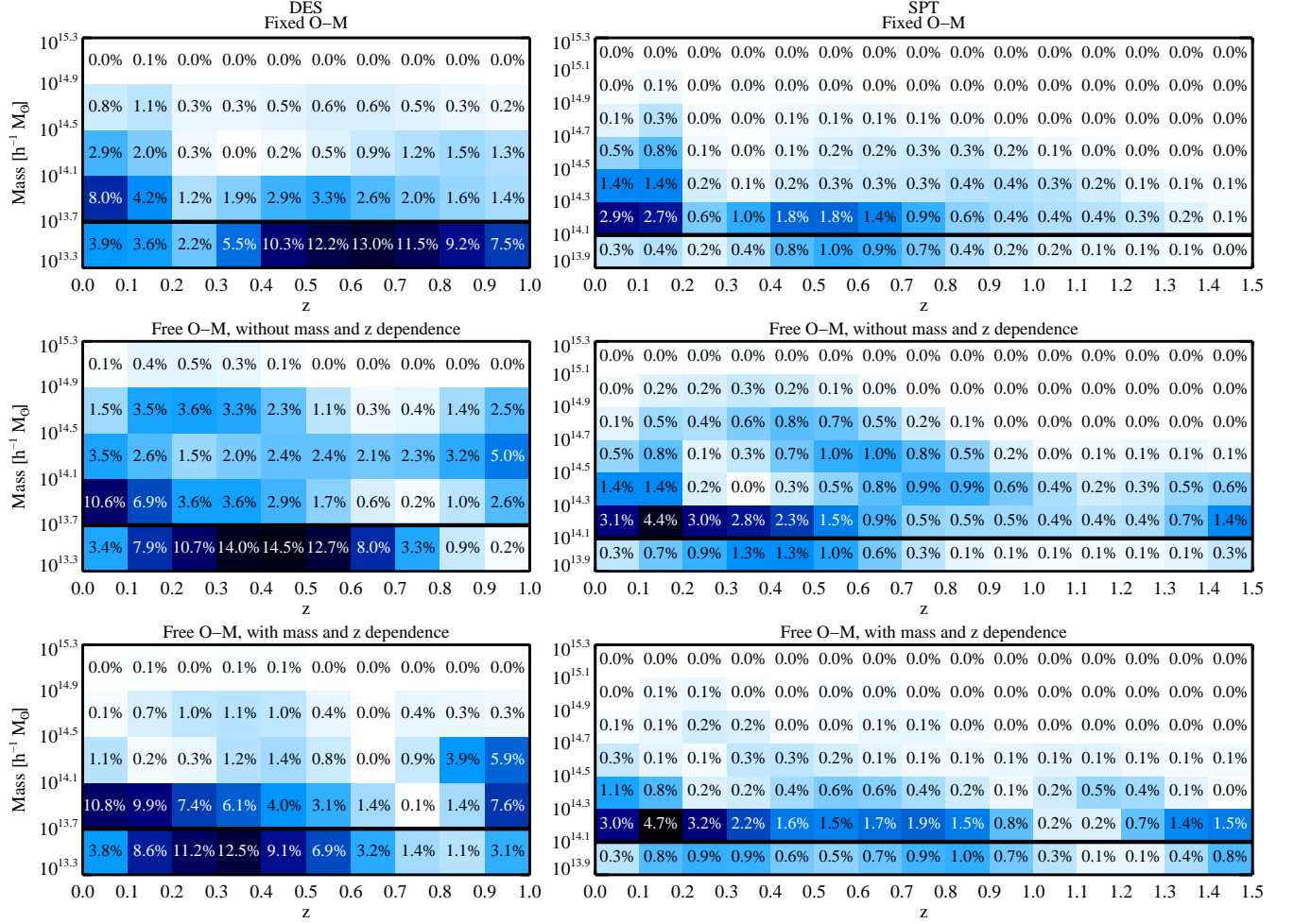
FIG. 4.— Relative importance of f_i in different mass (vertical axes) and redshift (horizontal axes) bins for a DES-like survey (*left*) and an SPT-like survey (*right*). The number in each bin reflects the percentage improvement in the FoM that results from tightening the prior on the mass function in that bin (from 10% to 1% for DES and from 30% to 3% for SPT), and the shading scales linearly with the number. The top panels correspond to a fixed observable–mass relation with known mean and scatter; the middle panels correspond to a free observable–mass relation in which the mean and scatter are self-calibrated but are not mass or redshift dependent; and the bottom panels correspond to a free observable–mass relation that has mass- and redshift-dependent mean and scatter specified by the parameters of Equation (7). The horizontal thick line shows the observable threshold; the low-mass bins near this threshold are the most important because of the high cluster counts in these bins. The patterns in the redshift dimension are largely determined by a combination of the CMB prior, the clusters counts, and the degeneracy between f_i and scatter.

which only $\ln M_0$ and B_0 of Equation (7) are free to vary. In this case, the observable–mass distribution has a free mean and a free scatter, but these have no mass or redshift dependence. Lastly, in the bottom panels, all eight parameters in Equation (7) are free to vary so that the observable–mass relation may have significant mass and redshift dependence.

One common trend for these different assumptions on the mass–observable relation is that the low-mass bins tend to be more important than the high-mass bins when calibrating the mass function. Because of the steepness of the mass function, the low-mass bins contain more statistical power than the high-mass bins. In addition, because of the scatter in the observable–mass distribution, accurate predictions at the low mass end are needed to account correctly for a potentially large number of high-M_{obs} clusters that up-scatter from relatively low true masses.

In the case of the fixed observable–mass relation, the pattern in redshift is driven by both the assumed CMB prior on cosmological parameters and the cluster counts. With the CMB prior, lowest-redshift bins tend to provide the most complementary information. Consequently, the low-redshift bins

are highlighted as particularly important to the calibration of the theoretical predictions. On the other hand, the low-redshift bins have small counts due to the small volume (in our DES-like survey, the peak in the number of clusters per redshift interval occurs near $z \approx 0.7$); therefore, some of the higher-redshift bins also have a strong influence on the dark energy constraints.

Comparing a DES-like survey and an SPT-like survey, we find that the lowest-redshift bins above the observable threshold (shown as a horizontal thick line) are highlighted in both cases. On the other hand, in the case of DES, the mass bins below the observable threshold are also highlighted because of the large scatter. These bins correspond to the greatest halo counts, and the up-scattering of halos from these bins will be a significant fraction of the cluster sample. Therefore, in the presence of a large scatter, the accuracy of the mass function below the observable threshold will impact the dark energy constraints. We also find that this pattern depends slightly on the binning in M_{obs}; when the observational bin is very wide, the importance of the mass function below the observable threshold is less significant because the up-scattering of

halos near the threshold is not well resolved. However, as we have already discussed, it is most beneficial to bin finely in observations if the observable–mass scatter can be understood even in a parameterized manner, because this results in maximum cosmological information.

The other two rows shown in Figure 4 allow for additional parameter freedom in the observable–mass relation that must be calibrated from the survey data itself. For each survey assumption, the structure in the two lower panels of Figure 4 is more complex than in the top panel. One general difference is that the low-redshift bins are relatively more important when the observable–mass relation must be calibrated, and this occurs primarily for two reasons. First, the low-redshift bins provide complementary information to the CMB prior. Second, the degeneracy between the mass function and the scatter is stronger at low redshift because the mass function is a shallower function at low redshift.

As a result, it is most important to set stringent priors on the mass function at low redshift and low mass. The relative importance of low-mass bins depends on the scatter of the observable–mass distribution. In the most general case of a mass- and redshift-dependent observable–mass distribution, new parameter degeneracies emerge but the general pattern is altered only slightly.

5. CONCLUSIONS

We have studied the impact of theoretical uncertainties in predictions of the halo mass function and halo bias on dark energy constraints from upcoming cluster surveys. Our primary conclusions are as follows.

1. Inaccuracy in the shape of the mass function at the 20% level (similar to the current state of theoretical uncertainty) can lead to significant systematic errors in dark energy parameter inferences.

2. For near-term cluster surveys like DES, the mass function must be predicted at approximately 1% accuracy to avoid more than 10% degradation in the resulting dark energy FoM. Similarly, the halo bias should be predicted with a precision of approximately 5%. The current state of uncertainty in these functions could decrease the FoM of a DES-like survey by a factor of ~ 2. Requirements are generally less restrictive for SZ and X-ray efforts.

3. A future optical survey over a significant fraction of the sky, like LSST, will require the most stringent constraints on the theoretical predictions. In this case, the mass function and halo bias must be computed with an accuracy of $\sim 0.5\%$ and $\sim 1\%$ respectively in order to guarantee that the theoretical uncertainty in these quantities is a negligible contributor to the dark energy error budget. This represents a practical limit to the accuracy with which these quantities will need to be computed in order to interpret future survey data.

4. Precise prediction of the mass function at the low masses close to the observable threshold and low redshifts that will be used in the analysis of survey data is the most beneficial to improve dark energy constraints.

We have considered the influence of theoretical uncertainties in comparison only to statistical errors on forthcoming measurements; however, additional systematic errors that will be present at different levels in different types of cluster surveys will make the demands on theoretical mass functions somewhat less restrictive. For example, the systematic errors due to inaccurate modeling of the observable–mass distribution (e.g. non-Gaussian scatter, Cohn et al. 2007; Shaw et al. 2009), an inaccurate calibration of completeness or false detection rate, and large errors in photometric redshift estimates will all increase errors on dark energy parameters and reduce the relative influence of uncertainties in predicted halo mass functions and biases. The requirements we advocate are relevant when these other known sources of error do not dominate the error budget. A robust interpretation of our calculations is that the requirements we quote render the error due to inaccurate predictions of halo abundances unimportant, but less stringent requirements may be adequate when systematic errors are large. However, we did show that the theoretical requirement is relatively insensitive to knowledge of the observable–mass relation when this relation is well described by a Gaussian distribution. This suggests that the requirements may not be considerably loosened if systematics are moderate.

Dark matter simulations currently have $\sim 5\%$ uncertainties in predicting mass functions for fairly standard cosmological models, even when the dark energy is *fixed* to a cosmological constant (Tinker et al. 2008). To achieve sub-percent level predictions in dark matter simulations requires a more careful consideration of systematics in halo finding (Heitmann et al. 2005) and initial conditions (Crocce et al. 2006), as well as a more careful study of the non-universality of the halo mass function (Tinker et al. 2008; Robertson et al. 2009). This will require simulations of a wider range of cosmological models that go beyond the standard cold dark matter with cosmological constant model. Reasonably exhaustive simulation programs are beginning (e.g. Desjacques et al. 2009; Maggiore & Riotto 2009b; Lam & Sheth 2009; Martino et al. 2009; Pillepich et al. 2010; Casarini et al. 2009; Jennings et al. 2010; Grossi et al. 2009; Alimi et al. 2010). Even more challenging still will be understanding the systematic uncertainties induced by baryonic physics. Recent studies have shown that these effects can result in significant deviations in the halo number density and that these effects are a function of the halo mass scale (Rudd et al. 2008; Stanek et al. 2009). Although meeting the stringent constraints we advocate may be quite challenging, pursuing further accuracy in theoretical predictions along with controlling various systematics in clusters surveys promises to continue to improve our knowledge of dark energy.

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